

Kumon Mathematics Program

Level M

Points and Lines
Circles, Loci, and Regions
Trigonometric Functions and Inequalities
Addition Formulas
Laws of Sines and Cosines
Area of Triangles

Student Name: _____

Starting Date: _____

Completion Date: Your Goal _____

Jan Apr July Oct
Feb May Aug Nov
Mar June Sep Dec

The goal is based upon your individual abilities. It takes into account the number of pages you are currently able to complete per day and the number of repetitions necessary to ensure mastery of the worksheets.

Goals of Level M

The goal of Level M is for you to develop the skills of working with geometric figures and equations, trigonometric ratios, and functions. You will first study about points and lines, learning about concepts such as internal division, external divisions, equations of lines, distance of a point to a line, and more. You will progress to study topics such as circles, loci, and regions, followed by the basics of trigonometric functions, trigonometric ratios, evaluating trigonometric expressions, and solving trigonometric equations. You will then graph trigonometric functions and learn to use the Addition Formulas. To conclude Level M, you will study the Laws of Sines and Cosines, as well as determine the area of triangles.

Contents of Level M

Worksheet Number	Section	Worksheet Number	Section
1 - 30	Points and Lines	121 - 130	Trigonometric Equations
31 - 50	Circles	131 - 140	Graphs of Trigonometric Functions
51 - 70	Loci	141 - 150	Trigonometric Inequalities
71 - 80	Regions	151 - 180	Addition Formulas
81 - 100	Trigonometric Ratios	181 - 190	Laws of Sines and Cosines
101 - 120	Properties of Trigonometric Functions	191 - 200	Area of Triangles

Features of Level M

M91a Trigonometric Ratios 2 KUMON® M 91

100% 90% 80% 70% 60% 50% 40% 30% 20% 10% 0%

As shown in the diagram, draw a semicircle with the center at origin O and radius r , and let A be the point $(r, 0)$. Place point $P(x, y)$ on the circumference of this semicircle and let $\angle AOP = \theta$.

Then, the following equalities are true:

$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}, \quad \tan \theta = \frac{y}{x}$$

Using the semicircle with radius 2, find the values of $\sin 60^\circ$, $\cos 60^\circ$ and $\tan 60^\circ$.

[Sol] Placing point P as shown in the diagram, point P is $(1, \sqrt{3})$.

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

1. Using the semicircle with radius $\sqrt{2}$, find the values of $\sin 45^\circ$, $\cos 45^\circ$ and $\tan 45^\circ$.

You will begin Level M by studying points and lines. M1-30 introduce topics such as distance formula, internal and external division formula, and the midpoint formula, with problems involving the coordinates of points.

You will also study equations of straight lines. This will include finding the equations of straight lines that satisfy certain conditions and studying relationships between straight lines (parallel, perpendicular lines, intersection, etc.).

In Worksheets 31-50, you will learn the equations of circles, properties of circles, and the equations of tangent lines to circles. You should get into the habit of drawing graphs in these sets so that you are prepared for upcoming topics in Level M.

M131a Graphs of Trigonometric Functions KUMON® M 131

100% 90% 80% 70% 60% 50% 40% 30% 20% 10% 0%

Graph of $y = \sin \theta$

Let the point of intersection of the terminal side of θ and the unit circle be P . The coordinates of P are $(\cos \theta, \sin \theta)$. Using this, it is possible to draw the graph of $y = \sin \theta$.

1. Draw the graph of $y = \sin \theta$ for $0 < \theta < 2\pi$.

[Sol]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	2π
$y = \sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	0

Graph of $y = \cos \theta$

1. Draw the graph of $y = \cos \theta$ for $0 < \theta < 2\pi$.

[Sol]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	2π
$y = \cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	1

Graph of $y = \tan \theta$

1. Draw the graph of $y = \tan \theta$ for $0 < \theta < 2\pi$.

[Sol]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	2π
$y = \tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	Undefined	-1	0

Graph of $y = \cot \theta$

1. Draw the graph of $y = \cot \theta$ for $0 < \theta < 2\pi$.

[Sol]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	2π
$y = \cot \theta$	Undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	Undefined	1	$\sqrt{3}$	Undefined	-1	Undefined

Graph of $y = \sec \theta$

1. Draw the graph of $y = \sec \theta$ for $0 < \theta < 2\pi$.

[Sol]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	2π
$y = \sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	Undefined	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-1	$-\frac{2}{\sqrt{3}}$	Undefined	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	1

Graph of $y = \csc \theta$

1. Draw the graph of $y = \csc \theta$ for $0 < \theta < 2\pi$.

[Sol]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	2π
$y = \csc \theta$	Undefined	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-1	$-\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	Undefined

Graph of $y = \tan \theta$ and $y = \cot \theta$

1. Draw the graph of $y = \tan \theta$ and $y = \cot \theta$ for $0 < \theta < 2\pi$.

[Sol]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	2π
$y = \tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	Undefined	-1	0
$y = \cot \theta$	Undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	Undefined	1	$\sqrt{3}$	Undefined	-1	Undefined

Graph of $y = \sec \theta$ and $y = \csc \theta$

1. Draw the graph of $y = \sec \theta$ and $y = \csc \theta$ for $0 < \theta < 2\pi$.

[Sol]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	2π
$y = \sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	Undefined	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-1	$-\frac{2}{\sqrt{3}}$	Undefined	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	1
$y = \csc \theta$	Undefined	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-1	$-\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	Undefined

You will solve problems on loci in Worksheets 51-70, including quadratic curves such as ellipses, hyperbolas, and parabolas.

You will learn to graph regions representing various inequalities in Worksheets 71-80.

Following Loci and Regions, you will be studying trigonometric functions. Worksheets 81-130 cover the basics of trigonometric functions such as trigonometric ratios, fundamental formulas for trigonometric functions, 30° - 60° - 90° triangles, and 45° - 45° - 90° triangles. After an introduction to general angles and radius vectors, you will calculate the trigonometric values of angles, work with radian measures, simplify trigonometric expressions, and solve trigonometric equations. Be sure to study all the examples in these sets and the rest of Level M very carefully.

In Worksheets 131-140, you will learn to graph basic trigonometric functions and should become very familiar with these basic curves. You will also solve trigonometric inequalities and obtain the maximum and minimum values of trigonometric functions in Worksheets 141-150.

Worksheets 151-180 feature the Addition Formulas. You will learn to solve various trigonometric equations and inequalities by applying these formulas. It is very important that you memorize the formulas, or can easily deduce the formulas that should be used.

To conclude Level M, you will study applications of trigonometry to triangles where you will learn the Laws of Sines and Cosines to determine lengths and angles for various triangles in Worksheets 181-190, followed by determining the area of different triangles in Worksheets 191-200.

Instructor's Comments