

# Kumon Mathematics Program

## Level N

**Arithmetic and Geometric Sequences**  
**Infinite Sequences and Series**  
**Limits of Functions**  
**Differentiation**

Student Name: \_\_\_\_\_

Starting Date: \_\_\_\_\_

Completion Date: Your Goal \_\_\_\_\_

The goal is based upon your individual abilities. It takes into account the number of pages you are currently able to complete per day and the number of repetitions necessary to ensure mastery of the worksheets.

Jan	Apr	July	Oct
Feb	May	Aug	Nov
Mar	June	Sep	Dec

### Goals of Level N

In Level N, you will begin by studying various new topics such as sequences, series, recurrence relations, and mathematical induction. Building on your knowledge of concepts learned in Level L, you will progress through the second half of Level N studying more advanced topics in limits of functions and differentiation. Level N aims to develop your ability to work with quadratic inequalities, sequences, series, limits, and differentiation, in preparation for Level O.

### Contents of Level N

Worksheet Number	Section	Worksheet Number	Section
1 - 10	Arithmetic Sequences	71 - 90	Infinite Geometric Sequences and Series
11 - 20	Geometric Sequences	91 - 100	Infinite Series
21 - 40	Various Sequences	101 - 120	Limits of Functions
41 - 50	Recurrence Relations	121 - 130	Limits of Trigonometric Functions
51 - 60	Mathematical Induction	131 - 140	Continuous and Discontinuous Functions
61 - 70	Infinite Sequences	141 - 200	Differentiation

# Features of Level N

**N22b**

Given two sequences  
 $(a_n)$ ,  $a_1, a_2, a_3, \dots, a_n$   
 $(b_n)$ ,  $b_1, b_2, b_3, \dots, b_n$   
 rewrite the following equalities using  $\Sigma$

(1)  $(a+b)_1 + (a+b)_2 + (a+b)_3 + \dots + (a+b)_n$   
 $\rightarrow (a_1 + b_1) + (a_2 + b_2) + (a_3 + b_3) + \dots + (a_n + b_n)$   
 $\rightarrow (a_1 + a_2 + a_3 + \dots + a_n) + (b_1 + b_2 + b_3 + \dots + b_n)$

(2)  $\sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$

(3)  $\sum_{k=1}^n (ca_k) = c \sum_{k=1}^n a_k$  ( $c$  is a constant)

(4)  $\sum_{k=1}^n ca_k = c \sum_{k=1}^n a_k$

From the above, the following equalities are true:

**Summation Properties**

$\sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$ ,  $\sum_{k=1}^n ca_k = c \sum_{k=1}^n a_k$  ( $c$  is a constant)

Find the following sums:

(1)  $\sum_{k=1}^n (4k+2) =$

(2)  $\sum_{k=1}^n (-2k+4) =$

(3)  $\sum_{k=1}^n (3k+5) =$

The first topic in Level N is Sequences. In Worksheets 1-40, you will learn about Arithmetic and Geometric Sequences and how to derive  $a_n$ , the general term of a sequence, and  $S_n$ , the sum of the first  $n$  terms. You will also learn about the summation symbol,  $\Sigma$  (sigma), and sequences of differences.

In Worksheets 41-60, you will apply your knowledge of sequences and learn about recurrence relations and mathematical induction.

While studying Infinite Sequences in Worksheets 61-80, you will learn how to determine whether a given sequence converges to a limit value, diverges to positive or negative infinity, or oscillates. In this section, you will practice calculating the limits of sequences.

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Limits of Trigonometric Functions

100% 100% 100% 100% 100% 100%

Find the following limits:

(1)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \lim_{x \rightarrow 0} \frac{1 - \cos x + 0 + 0 + 0 + 0}{x + 0 + 0 + 0 + 0 + 0}$   
 $= \lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$   
 $= \lim_{x \rightarrow 0} \frac{0 + 0 + 0 + 0 + 0 + 0}{1 + 0 + 0 + 0 + 0 + 0}$   
 $= \lim_{x \rightarrow 0} \frac{0}{1} = 0$

(2)  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{0 + 0 + 0 + 0 + 0 + 0}{1 + 0 + 0 + 0 + 0 + 0}$   
 $= \lim_{x \rightarrow 0} \frac{\sin x}{x}$   
 $= \lim_{x \rightarrow 0} \frac{0 + 0 + 0 + 0 + 0 + 0}{1 + 0 + 0 + 0 + 0 + 0}$   
 $= \lim_{x \rightarrow 0} \frac{0}{1} = 0$

Following sequences, you will study the topic of Series in Worksheets 81-100. A series expresses the sum of all terms in a given sequence.

In Worksheets 101-130, you will find the limits of logarithmic functions, trigonometric functions, and other various functions.

Next, you will study and graph continuous and discontinuous functions in Worksheets 131-140. Here you will learn the Intermediate Value Theorem.

Level N concludes with a study of Differentiation. You will practice obtaining the first, second, and third derivatives of certain functions and end with the study of important theorems like the Rolle's Theorem and the Mean Value Theorem.

## Instructor's Comments

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