

# Kumon Mathematics Program

## Level J

**Factorization**  
**Irrational Numbers**  
**Quadratic Equations**  
**Remainder and Factor Theorems**  
**Proof of Equalities and Inequalities**

Student Name: \_\_\_\_\_

Starting Date: \_\_\_\_\_

Completion Date: Your Goal \_\_\_\_\_

Jan	Apr	July	Oct
Feb	May	Aug	Nov
Mar	June	Sep	Dec

The goal is based upon your individual abilities. It takes into account the number of pages you are currently able to complete per day and the number of repetitions necessary to ensure mastery of the worksheets.

## Goals of Level J

The main goal of this level is for you to master the algebraic calculation skills that are essential for studying high school level math. Such skills include: simplifying, evaluating and factoring algebraic expressions, working with fractional and irrational expressions, and solving quadratic and simultaneous equations.

## Contents of Level J

Worksheet Number	Section	Worksheet Number	Section
1-10	Expansion of Polynomial Products	131-140	Root-Coefficient Relationship
11-60	Factorization	141-150	Simultaneous Equations
61-70	Fractional Expressions	151-160	Dividing Polynomials
71-90	Irrational Numbers	161-170	Remainder Theorem
91-110	Quadratic Equations	171-180	Factor Theorem
111-120	Quadratic Equations and Complex Numbers	181-190	Proof of Identities and Equalities
121-130	Discriminant	191-200	Proof of Equalities and Inequalities

# Features of Level J

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Factorization 3

Time: \_\_\_\_\_ Date: \_\_\_\_\_ Name: \_\_\_\_\_

100%	90%	80%	70%	60%

Factor the following expressions as shown in the example.

Ex.  $x^2 + (3y+4)x + (2y^2 + 5y + 3)$   
 $= x^2 + (3y+4)x + (y+1)(2y+3)$   
 $= (x+y+1)[x+(2y+3)]$   
 $= (x+y+1)(x+2y+3)$

First factor the quadratic  $(2y^2 + 5y + 3)$ .

(1)  $x^2 + (3y+5)x + (2y^2 + 7y + 6)$

(2)  $x^2 - (3y+4)x + (2y^2 + 3y - 5)$

(3)  $x^2 + (3y+6)x + (6y^2 + 13y + 5)$

In Worksheets 11-60, you will learn many different formulas and methods of factoring complex algebraic expressions.

Applying your knowledge of square roots from Level I, you will solve exercises with irrational numbers and expressions in Worksheets 71-90.

Imaginary numbers will be introduced in Worksheets 111-115. You will then begin solving quadratic equations with complex number solutions. In Worksheets 121-130, you will learn to calculate the *Discriminant*, which you will use to determine what types of solutions a quadratic equation will have: 2 different real number solutions, a repeated solution or 2 different complex number solutions.

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J 121 a

Discriminant

Time: \_\_\_\_\_ Date: \_\_\_\_\_ Name: \_\_\_\_\_

100%	90%	80%	70%	60%

Quadratic Formula

Solving the equation  $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Solve the following equations by using the quadratic formula.

(1)  $x^2 + 8x + 15 = 0$       (4)  $x^2 - 4x + 4 = 0$

(2)  $x^2 + 8x + 16 = 0$       (5)  $x^2 - 4x + 5 = 0$

(3)  $x^2 + 8x + 17 = 0$       (6)  $x^2 - 4x + 2 = 0$

Looking at the results of the exercises above, we can see that there are three types of solutions:

- 2 different real number solutions, in exercises (1) and
- a repeated solution, (only 1 solution), in exercises (2) and
- 2 different complex number solutions, in exercises (3) and

In Worksheets 151-160, you will use long division to divide polynomials. You should master this skill, as you will need it to progress smoothly through the *Remainder Theorem* and *Factor Theorem* sets that follow.

You will finish this level by conducting proofs of equalities and inequalities. This is a very important skill that you will carry on in your studies of geometry and trigonometry.

## Instructor's Comments

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