Kumon Mathematics Program

Level L

Logarithmic Functions
Limits and Derivatives
Maxima and Minima
Indefinite and Definite Integrals
Integral Applications

Student Name: _____

Starting Date: _____

Completion Date: Your Goal

Jan	Apr	July	Oct
Feb	May	Aug	Nov
Mar	June	Sep	Dec

The goal is based upon your individual abilities. It takes into account the number of pages you are currently able to complete per day and the number of repetitions necessary to ensure mastery of the worksheets.

Goals of Level L

The first goal of Level L is for you to calculate and express logarithms, graph logarithmic functions, and solve logarithmic equations & inequalities. You will then be led into the beginning of calculus, studying basic differentiation and integration. Level L aims to strengthen your skills in graphing functions, obtaining relative maxima & relative minima, and maxima & minima, solving equations & inequalities, and analyzing applications of integration including areas, volumes, velocity and distance.

Contents of Level L

Worksheet Number	Section	Worksheet Number	Section
1-10	Logarithmic Functions	101-110	Applications to Equations and Inequalities
11-20	Graphs of Logarithmic Functions	111-120	Indefinite and Definite Integrals
21-30	Logarithmic Equations and Inequalities	121-140	Definite Integrals
31-40	Modulus Functions	141-160	Areas
41-50	Limits and Derivatives	161-170	Volumes
51-60	Tangent Lines	171-180	Velocity and Distance
61-100	Relative Maxima & Minima, Maxima & Minima	181-200	Summary of Derivatives and Integrals

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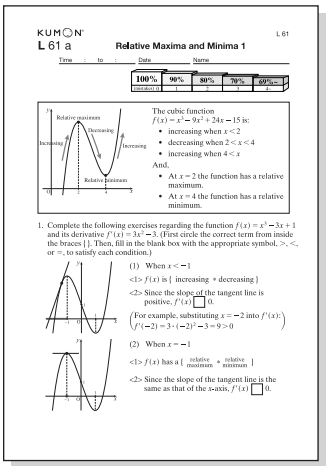
Features of Level L


KUMON
L 61 a
Relative Maxima and Minima 1
L 61
Time : to : Date Name
100% 90% 80% 70% 60%
The cubic function $f(x) = x^3 - 9x^2 + 24x - 15$ is:
- increasing when $x < 2$
- decreasing when $2 < x < 4$
- increasing when $4 < x$

And,
- At $x = 2$ the function has a relative maximum.
- At $x = 4$ the function has a relative minimum.

1. Complete the following exercises regarding the function $f(x) = x^3 - 3x + 1$ and its derivative $f'(x) = 3x^2 - 3$. (First circle the correct term from inside the braces { }. Then, fill in the blank box with the appropriate symbol, >, <, or =, to satisfy each condition.)

(1) When $x < -1$

<1> $f(x)$ is { increasing • decreasing }

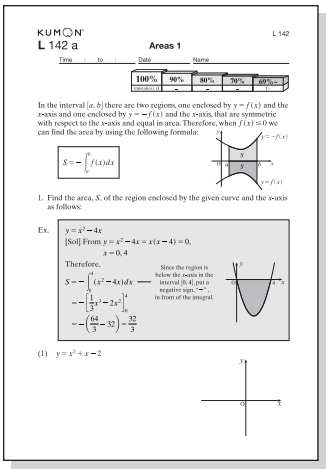
<2> Since the slope of the tangent line is positive, $f'(x)$ ☐ 0.

(For example, substituting $x = -2$ into $f'(x)$: $f'(-2) = 3 \cdot (-2)^2 - 3 = 9 > 0$)

(2) When $x = -1$

<1> $f(x)$ has a { relative maximum • relative minimum }

<2> Since the slope of the tangent line is the same as that of the x-axis, $f'(x)$ ☐ 0.


KUMON
L 142 a
Areas 1
L 142
Time : to : Date Name
100% 90% 80% 70% 60%
In the interval $[a, b]$ there are two regions, one enclosed by $y = f(x)$ and the x-axis and one enclosed by $y = -f(x)$ and the x-axis, that are symmetric with respect to the x-axis and equal in area. Therefore, when $f(x) \leq 0$ we can find the area by using the following formula:

$S = -\int_a^b f(x)dx$

1. Find the area, S, of the region enclosed by the given curve and the x-axis as follows:

Ex. $y = x^2 - 4x$

[Sol] From $y = x^2 - 4x = x(x - 4) = 0$, $x = 0, 4$

Therefore,

$S = -\int_0^4 (x^2 - 4x)dx = -\left[\frac{1}{3}x^3 - 2x^2\right]_0^4 = -\left(\frac{64}{3} - 32\right) = \frac{32}{3}$

Since the region is below the x-axis in the interval [0, 4], put a negative sign, "−", in front of the integral.

(1) $y = x^2 + x - 2$

You will begin Level L by studying logarithmic functions. In Worksheets 1-30, you will learn the properties of logarithms, solve equations and inequalities, and draw graphs of logarithmic functions.

Worksheets 31-40 feature *Modulus Functions*. In this set, while drawing graphs of functions with absolute values, be sure to consider all the cases under the given domain.

Set 41-50 marks the beginning of your calculus studies. This section introduces the *average rate of change*, *differential coefficients* and *derivatives*, and the *rules of differentiation*.

In Worksheets 51-60, you will use differentiation to obtain the slopes and equations of tangent lines.

Worksheets 61-100 focus on relative maximum & minimum values, and maximum & minimum values. In these exercises, as well as in the applications of set 101-110, it is very important that you learn to create *variation tables* displaying the increasing and decreasing information, and relative extreme values of functions.

The learning focus of Worksheets 111-180 is integration. You will learn the *Properties of Indefinite and Definite Integrals* and use them to evaluate integrals. In L141-160, you will use integration to determine the areas of regions enclosed by given curves and/or lines. You should always sketch a graph and remember that areas have a positive value. In Worksheets 161-170, you will determine the volume of solids of revolution, and in Worksheets 171-180, you will solve application exercises involving velocity and distance.

To conclude Level L, you will review differentiation and integration through a summary of exercises in L181-200.

Instructor's Comments

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